Note on Modern Mathematical Methods

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Abstract

This is the lecture note for Modern Mathematical Methods lectures of 2023 Spring semester in University of Science and Technology of China given by Prof. Sen Hu. Due to certain reasons, this note may not be updated in time every week.

Contents

0 Introduction

0.1 Outline

0.2 References

For gauge theory and classical mechanics, need to take :

Vladimir Arnold, Mathematical Methods in Classical Mechanics

Theodore Frenkel, The Geometry of Physics

For quantum theory, need to take:

Stephen Weinberg, Quantum Field Theory.

Yellow book: Conformal Field Theory.

1 Classical Theory

1.1 Classical Mechanics

1.1.1 Newton Mechanics

Galilio's relativoty principle $\rightarrow$ Newton's 1^{st} law

Definition of event: classical spacetime in $\mathbb{R}^4$: $(t, \vec{x})$

Flaws of classical spacetime: ϵ can be small enough $\Rightarrow$ conflict with QM (*Heisenberg Constant*)

$\Rightarrow$ We need SR

Basically the essence of Newtonian: Galileo transformation keeps the spacetime coordinates: inertial frame invariant under poincare group action.

Poincare Group: shift and rotation

with rotation, inner product $\Rightarrow SO(n)$ (notice that $n+1$ is the dimension of spacetime)

Newton's 2^{nd} law:

Fermat: derivatives $\rightarrow$ velocity $\rightarrow^{derivatives}$ acceleration

Galileo: Acceleration changes the state of object.

One-body: Configuration space: $\mathbb{R}^{3n}/\{\vec{x_i} = \vec{x_j},\ i \neq j\}$: elliminate the effect of mechanical interaction of objects; And still the spacetime stays invariant of the group action of diffential poincare transformation

Many-body: $(\vec{x_1}, ...\vec{x_n}) \in \mathbb{R}^{3n}$

$$m_i \frac{d^2\vec{x_i}}{dt^2} = \sum_{r \neq i} \left(-\frac{m_i m_j}{|x_i - x_j|^3} (\vec{x_i} - \vec{x_j})\right) \tag{1}$$

Consider why do we need square in the denominator?: Since the real world (classical) is 3 dimensional (Mar.6, 2023)

1.2 SR